JURNAL TEKNIK SIPIL
MACCA

Vol. 10, No. 2, JUNI 2025
E-ISSN: 2720-9199
DOI: https://doi.org/10.33096/tpkgk930

Analysis of Single Pile Foundation Bearing Capacity Using A Bayesian Approach

I Wayan Ariyana Basoka *

Program Studi Teknik Sipil, Universitas Warmadewa, Denpasar
*basokaariyana@warmadewa.ac.id

Diajukan: 29 Mei 2025, Revisi: 03 Juni 2025, Diterima: 30 Juni 2025

Abstract

Pile foundations are essential in geotechnical engineering, especially when strong soil layers are too deep for shallow foundations. Besides supporting vertical loads, they must also resist lateral forces from wind and earthquakes. Accurately estimating pile bearing capacity is crucial for structural safety and cost efficiency. Conventional methods, such as those using SPT and CPT results, are mostly deterministic and often ignore the uncertainty in soil properties—an issue especially relevant in tropical regions like Indonesia with highly variable subsurface conditions.

This study applies a Bayesian probabilistic approach to analyze pile bearing capacity using CPT data from Tumbak Bayuh, Bali. Typical soil parameters from literature are used as priors and updated with site-specific data. The results indicate that Bayesian inference offers more realistic and robust estimates than traditional methods by incorporating uncertainty and allowing continuous refinement as new data are added.

The study also introduces a Bayesian model to assess how sample size affects parameter estimation, comparing it with frequentist approaches such as least squares and maximum likelihood. Findings highlight the strength of Bayesian updating in capturing uncertainty, offering a systematic and adaptive framework for pile analysis—especially beneficial in data-scarce or variable geotechnical environments.

Keywords: pile foundation, bearing capacity, bayesian

1. INTRODUCTION

Pile foundations are one of the essential elements in geotechnical engineering, functioning to transfer loads from a structure to soil or rock layers with adequate bearing capacity. These foundations are typically used when competent soil strata are located at depths that are beyond the reach of shallow foundations. In such conditions, pile foundations become the primary option to ensure the stability and safety of structures, especially for high-rise buildings, bridges, towers, and other critical infrastructure.

In addition to supporting vertical loads from the structure's self-weight, pile foundations must also perform reliably under lateral loads induced by environmental factors such as wind and earthquakes. Therefore, the design and analysis of pile bearing capacity must be conducted with great care to prevent structural failure, which could result in serious public safety hazards and significant economic losses. Accurate estimation of pile bearing capacity also contributes to cost-efficient construction by avoiding overly conservative or, conversely, underdesigned solutions.

Over the past few decades, various empirical, semi-empirical, and analytical methods have been developed to estimate the bearing capacity of pile foundations. These methods are generally based on geotechnical parameters such as soil cohesion, internal friction angle, and field test data, including the Standard Penetration Test (SPT) and Cone Penetration Test (CPT). However, most of these approaches remain deterministic in nature, assuming that soil parameters are known with certainty. In reality, geotechnical parameters often exhibit significant variability and uncertainty due to natural heterogeneity and limitations in field and laboratory data.

Uncertainty in geotechnical parameters can lead to substantial deviations between predicted and actual field performance. To address this issue, probabilistic approaches have been increasingly introduced in geotechnical analysis to account for variability and uncertainty. Among these, the Bayesian approach has gained considerable attention for its ability to integrate available data with prior knowledge, resulting in parameter estimates and output predictions that are more data-driven and realistic (Albert & Hu, 2019; Baecher, 2017; Basoka et al., 2023, 2024; Bozorgzadeh et al., 2019; Fornacon-Wood et al., 2022; Gelman et al., 2021; Kelly & Huang, 2015; Kruschke, 2021; Ueda, 2022; Wakefield, 2013; Zaidi et al., 2012).

Although the Bayesian approach offers advantages in terms of uncertainty quantification and dynamic information updating, its application in the analysis of pile bearing capacity remains relatively limited. Initial studies have shown promising results, yet further research is needed to evaluate its reliability in practical design contexts, particularly under tropical soil conditions such as those found in Indonesia, which are characterized by complex and highly variable geotechnical properties.

2. METHOD

a) Soil Parameter

Soil bearing capacity evaluation is commonly conducted through in-situ tests such as the Standard Penetration Test (SPT) and Cone Penetration Test (CPT). These tests provide practical and site-specific information about subsurface conditions. However, it is not uncommon for the results obtained from such field investigations to differ from the typical soil parameter values reported in previous studies or reference literature. Such discrepancies may arise due to local variations in soil type, depositional history, weathering processes, or testing conditions.

Table 1. Soil Parameters

Soil Type	CPT qc Range(MPa)	Unit Weight(kN/m³)	Cohesion (c)(kPa)	Friction Angle (φ)(degrees)
Soft Clay	0.2 – 1.0	15 – 18	15 – 30	5 – 15
Stiff Clay	1.0 – 4.0	17 – 20	30 – 100	15 – 25
Loose Silty Sand	1.0 – 5.0	16 – 19	0 – 5	25 – 30
Medium Dense Sand	5.0 – 10.0	18 – 20	0 – 5	30 – 35
Dense Sand	10.0 – 20.0	19 – 21	0 – 5	35 – 40

Soil Type	CPT qc Range(MPa)	Unit Weight(kN/m³)	Cohesion (c)(kPa)	Friction Angle (φ)(degrees)
Gravelly Sand	> 20.0	20 – 23	0 – 5	38 – 45
Organic Clay / Peat	< 0.5	10 – 15	5 – 15	5 – 10
Silty Clay	0.5 – 2.0	16 – 19	20 – 40	10 – 20

Table 1 presents a compilation of typical geotechnical parameters for various soil types, as documented by previous researchers. These values serve as a useful reference, especially when site investigation data are limited or when anomalies are observed in field test results. While they should not replace site-specific data, these typical parameter ranges can offer valuable context and initial estimates for engineering analysis.

In the context of Bayesian analysis, the typical values shown in Table 1 can serve as prior information or baseline assumptions. Bayesian methods rely on the integration of prior knowledge with observed data to produce updated (posterior) estimates. By using parameter ranges from established literature as priors, and combining them with data obtained from SPT or CPT, a more comprehensive and probabilistically sound assessment of soil bearing capacity can be achieved. This approach allows for the quantification of uncertainty and improves the reliability of geotechnical design, particularly when field data are sparse or exhibit high variability.

Moreover, the use of typical parameter values as priors in Bayesian analysis offers a systematic framework for incorporating engineering judgment and accumulated empirical knowledge into the design process. This is particularly beneficial in regions with limited geotechnical data or where testing conditions may not fully capture subsurface complexity. By calibrating these prior estimates with actual site investigation results, engineers can reduce the risk of underestimating or overestimating foundation performance. In turn, this enhances the robustness of the analysis and supports the development of more reliable and cost-effective geotechnical designs.

b) Pile foundation

The allowable bearing capacity of a pile foundation based on Cone Penetration Test (CPT) results, or sondir, is commonly determined through a combination of end bearing and shaft friction components. This method is widely used in geotechnical practice due to its direct use of in-situ data, which reflects the actual subsurface conditions with minimal disturbance. The empirical formula employed is as follows:

$$Q = (qc \,.\, Ab)/F1 + (JHP \,.\, O)/F2$$

In this equation:

qc represents the cone resistance, obtained from CPT measurements around the base of the pile, Ab is the cross-sectional area of the pile base, JHP denotes the total adhesion or skin resistance developed along the pile shaft, O is the perimeter of the pile shaft, $F1$ and $F2$ are safety factors for the bearing and friction components, typically taken as 3 and 5, respectively.

This approach provides a practical balance between simplicity and reliability. The use of separate safety factors for the two resistance mechanisms reflects the inherent variability and uncertainty associated with each. End bearing resistance is often more reliable due to concentration of stress at the pile tip, whereas shaft resistance is more sensitive to installation method, soil disturbance, and pile-soil interface conditions.

For bored piles, which are constructed by excavating a hole and then filling it with concrete (as opposed to being driven into the ground), the actual load-carrying capacity is generally lower than that of driven piles. This reduction is attributed to potential loosening of the soil during excavation, lack of compaction around the shaft, and imperfect concrete placement. To account for these differences, Craig (1994) recommends the use of reduction factors: one-third for end bearing capacity and one-half for shaft friction, relative to driven pile values. These factors are empirical but are widely accepted in geotechnical engineering practice, particularly in preliminary design stages.

The application of these empirical relationships must, however, be calibrated with local soil behavior and construction practices. In tropical regions like Indonesia, for instance, high variability in soil stratigraphy, presence of residual soils, and weathered rock layers may necessitate more detailed interpretation of CPT data and possible site-specific adjustments to the empirical coefficients or safety factors.

Furthermore, integration of probabilistic approaches such as the Bayesian method into the interpretation of CPT-based capacity estimations can enhance the reliability of the results by explicitly accounting for uncertainties in both soil parameters and measurement data. This is especially important for critical structures or sites with limited investigation data, where deterministic estimates may not adequately represent the range of possible performance outcomes.

c) Frequentist approach

Frequentist analysis is performed to obtain the shape parameter and scale parameter, which are the parameters of the Weibull distribution. The estimation method used in this analysis is using the least square method (LSM) and maximum likelihood estimate (MLE), as explained by Li et al. (2017). The linear regression method is the Least Squares Method (LSM). The slope of a simple linear regression determines the Weibull modulus. Rossi (2018) calculates presumptive probability distribution parameters from observable data using maximum likelihood estimate (MLE). Maximizing a likelihood function increases the probability that observed data matches the expected statistical model. The maximum likelihood estimate is where the likelihood function is most significant in the parameter space, Weibull parameter analysis employing frequentist inference and statistical software.

d) Bayesian Approach

On the other hand, geotechnical engineers are tasked with addressing the uncertainties that arise because of the constraints of inadequate knowledge (Baecher, 2017; Contreras et al., 2018; Phoon et al., 2022). These situations pertain to the probabilities of distinct occurrences. As mentioned earlier, the uncertainties are not easily addressed under the framework of Frequentist thinking but rather necessitate the use of Bayesian reasoning. Bayesian thinking pertains to the processes of judgment and belief. This phenomenon results in highly significant deductions even when the available data is limited (Baecher, 2017; Contreras et al., 2018).

A probability model is fitted to a collection of data using the Bayesian inference process, and the result is summarized by a probability distribution on the model's parameters and unobserved values like forecasts for future observations (Albert & Hu, 2019; Gelman et al., 2021). The posterior density is obtained by mere conditioning on the known value of the data y and applying the fundamental Bayes' rule of conditional probability as Eq. (3) below:

$$p(\theta|y) = \frac{p(\theta)p(y|\theta)}{p(y)} \quad (3)$$

Where $p(\theta)$ is the probability of hypothesis θ being true, despite the data, this is the prior probability of θ or the unconditional probability. The $p(y)$ is the probability of the data, regardless of the hypothesis, this is known evidence. The $p(y|\theta)$ is the probability of data y given that hypothesis θ is true known as the likelihood of data y conditional on hypothesis θ. The $p(\theta|y)$ is the probability of hypothesis θ given the data y, known as the posterior probability.

The utilization of the Bayesian inference model flow chart for statistical processing can be seen in Figure 1. Determining the random parameters mu_a and mu_b, which follow a normal distribution, is the initial step toward determining the deterministic parameters α and β. Subsequently, both α and β are utilized as inputs for the likelihood function, which follows the Weibull distribution. Finally, the posterior is computed based on the steps mentioned above. Subsequently, the procedure mentioned above was iterated, and the specimens were amplified utilizing Markov Chain Monte Carlo (MCMC). The process of conducting Bayesian analysis involves the initial establishment of priors, followed by the construction of the likelihood form, and ultimately culminating in the posterior computation, as delineated in Figure 1. The utilization of Python coding facilitates the implementation of the analysis process, which involves an iterative approach utilizing MCMC.

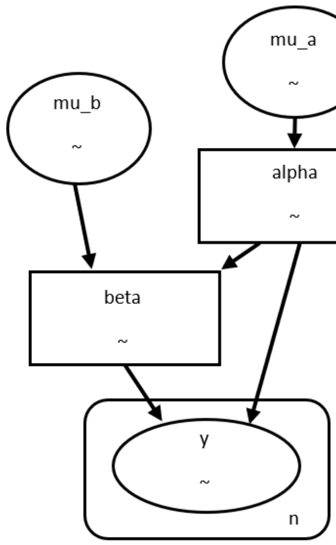

Figure 1. Framework of Bayesian inference model.

e) Hierarchical Bayesian Approach

Another method that will be used in this analysis is the Bayesian hierarchical approach. There are several benefits and advantages associated with the utilization of Hierarchical Bayesian modeling. Hierarchical Bayesian Models refer to a category of Bayesian statistical models that facilitate the representation and analysis of intricate data structures characterized by hierarchical relationships. These models include both individual-level and group-level information, facilitating the exchange of information across various levels of the hierarchy and resulting in more accurate and robust inferences (Albert & Hu, 2019; Bozorgzadeh et al., 2019; Britten et al., 2021; Feng et al., 2021; Gelman et al., 2021; Jiao et al., 2011; Ueda, 2022). Hierarchical Bayesian models are frequently employed across diverse sectors, including but not limited to education, sports, medicine, ecology, and finance. Hierarchical Bayesian modeling offers a versatile and comprehensible approach to expanding basic models of individual particle strength through the utilization of the Weibull distribution.

The hierarchical model is created by combining sub-models, and Bayes' theorem integrates the elements and incorporates all uncertainty. Markov Chain Monte Carlo (MCMC) techniques have helped develop and use Bayes' theorem, especially in hierarchical models (Allenby et al., 2005). **Error! Reference source not found.** is a flow chart of Bayesian hierarchical modeling. It almost resembles the usual Bayesian inference model, only here there is an additional hyperparameter s_a (standard deviation for mu_a) b0 (mean for mu_a), s_b (standard deviation for mu_b) b0 (mean for mu_b). In the prior data section, it is observed using random variables and observation parameters. Then the posterior is calculated based on the amount of data and data groups. The proposed model involves the analysis of Weibull parameters for each sample group (S, M, L) to gain insight into the impact of sample size on the parameters. The obtained results will be compared with frequentist inference to evaluate any observed changes.

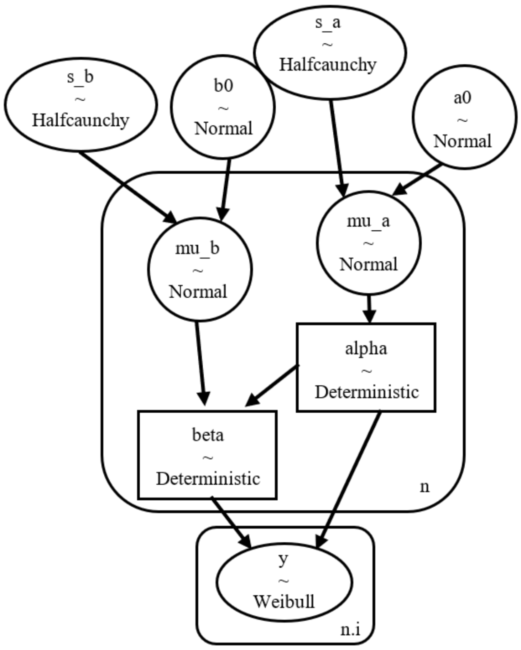

Figure 2. Framework of Hierarchical Bayesian inference model.

3. RESULTS AND DISCUSSIONS

a) CPT Results

The results of Cone Penetration Test (CPT) conducted at two different locations in the Tumbak Bayuh area, Kerobokan, North Kuta District, Badung Regency, Bali are presented in Figure 3. These CPT soundings were carried out to evaluate the subsurface soil conditions and to identify the depth at which soil layers exhibit sufficient bearing resistance to support foundation loads.

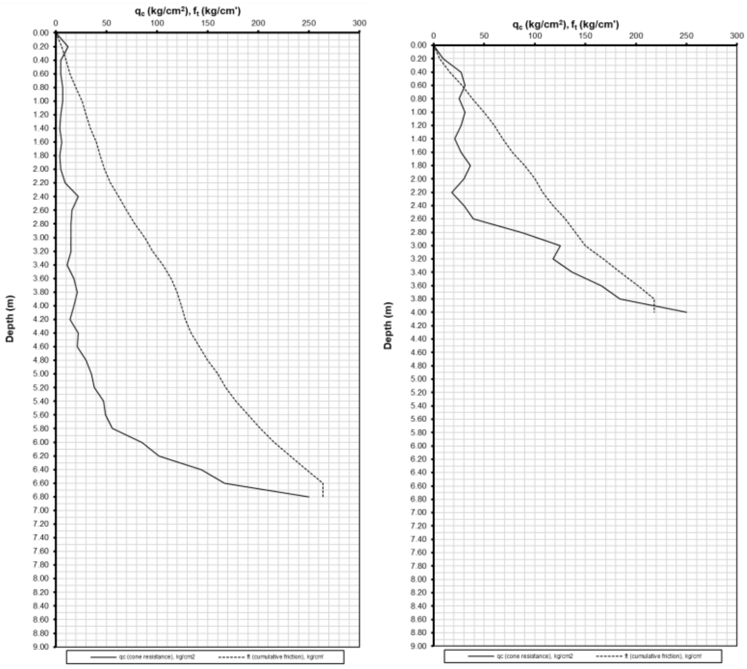

Figure 3. CPT Results

Based on the CPT profiles, it can be observed that the average depth to a competent soil layer—characterized by a significant increase in cone resistance values—ranges between 4 to 7 meters below the ground surface. This depth range is consistently identified across all five test points, indicating a relatively uniform subsurface stratigraphy in the investigated area. The soil strata encountered in this interval are predominantly classified as silty sand, or pasir kelanauan, based on Soil Behavior Type (SBT) charts interpreted from CPT data. This type of soil typically demonstrates moderate to high cone resistance, indicating improved strength and stiffness compared to the overlying weaker layers.

The upper soil layers, generally extending from the surface to depths of 3 to 4 meters, consist of softer, more compressible materials such as silty clay or clayey silt, which exhibit lower cone resistance values. These upper layers are not suitable for foundation support without improvement, due to their low bearing capacity and high compressibility. The presence of the denser silty sand layer at greater depths, however, provides a more favorable bearing stratum for deep foundations such as piles, or for shallow foundations with excavation to a deeper founding level.

In addition to identifying the depth of competent soil, the CPT data also provide valuable insight into the variability of subsurface conditions across the site. Although the general trend indicates a relatively consistent profile, minor variations in cone resistance values between locations suggest localized differences in soil density, gradation, or moisture content. These variations should be carefully considered during foundation design, particularly in determining pile length or the required depth of excavation, to ensure uniform structural performance and avoid differential settlement.

Furthermore, while silty sand generally offers better strength properties compared to clay-rich soils, its engineering behavior may still be sensitive to environmental conditions, such as groundwater fluctuations or cyclic loading due to seismic events. Given the site's location in a coastal and seismically active region, these factors must be incorporated into the design through rigorous geotechnical assessment. The integration of probabilistic analysis—such as Bayesian updating—with the CPT data allows engineers to explicitly account for uncertainties and site variability, providing a more reliable estimate of bearing capacity and a safer, more cost-efficient foundation solution.

b) Bayesian approach

Based on the analysis conducted, a comparison between the bearing capacity of single pile foundations calculated with and without the Bayesian approach is presented in Figure 4.

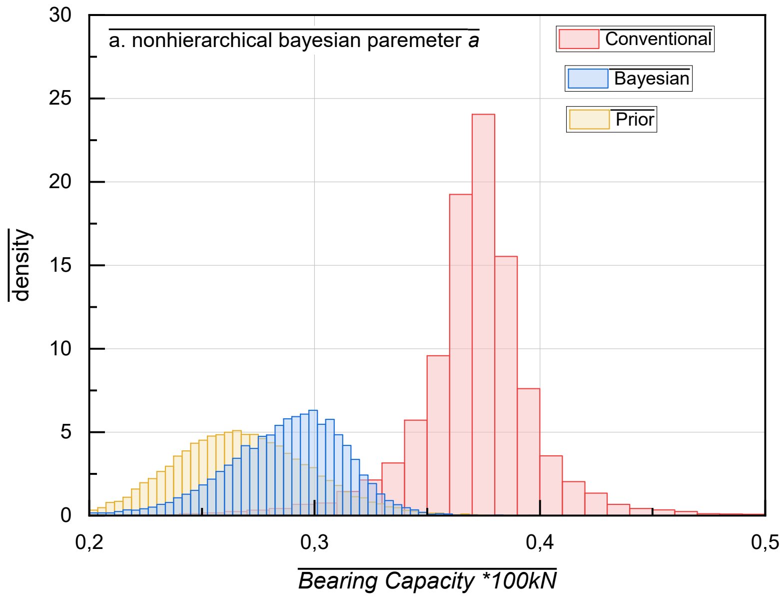

Figure 4. Bearing capacity of pile foundation

As shown in Figure 4, the bearing capacity estimation without using the Bayesian approach represents a purely deterministic analysis. This method does not account for prior information regarding typical soil strength parameters derived from previous studies. As a result, the outcome stands independently, relying solely on the current site investigation data, and does not incorporate any historical or contextual knowledge that might improve its reliability.

In contrast, the analysis incorporating the Bayesian approach explicitly considers uncertainty in the soil parameters by integrating prior knowledge from earlier research conducted on similar soil types. By applying prior probability distributions to model expected soil behavior—based on well-documented studies—the Bayesian method allows this historical information to influence and adjust the results obtained from current site data. This process results in an updated (posterior) estimate that better reflects both the observed data and the broader geotechnical understanding of the site conditions.

The incorporation of uncertainty through Bayesian inference is particularly valuable in geotechnical engineering, where soil properties are inherently variable and rarely known with complete certainty. Rather than treating the analysis as fixed and absolute, the Bayesian framework offers a dynamic and data-driven approach that adapts to new information. Consequently, this method improves the accuracy of pile capacity prediction, enhances confidence in the design, and contributes to safer and more cost-effective foundation solutions.

Furthermore, the Bayesian approach facilitates continuous updating of the soil parameter estimates as new data becomes available, making it a powerful tool for iterative design and decision-making processes. This adaptability is especially beneficial in complex or variable soil conditions, such as those often encountered in tropical regions like Bali, where soil heterogeneity and environmental factors can significantly impact foundation performance. By leveraging both prior knowledge and current measurements, engineers can achieve a more robust understanding of the subsurface conditions, ultimately leading to foundation designs that are not only safer but also optimized in terms of material usage and construction costs.

4. CONCLUSSIONS

The Cone Penetration Test (CPT) results from multiple locations in the Tumbak Bayuh area, Kerobokan, North Kuta District, Bali, reveal a relatively consistent subsurface profile with a competent soil layer predominantly composed of silty sand at depths between 4 to 7 meters. The upper layers consist of softer, compressible soils unsuitable for direct foundation support without improvement. The denser silty sand layer identified provides a suitable bearing stratum for pile foundations or deeper shallow foundations, though localized variations in soil properties highlight the importance of careful design consideration to prevent differential settlement.

The comparison between deterministic bearing capacity calculations and those incorporating a Bayesian probabilistic approach demonstrates the added value of integrating prior knowledge and quantifying uncertainty in geotechnical analysis. While deterministic methods rely solely on current site data, Bayesian updating combines historical soil parameter information with new measurements, resulting in more realistic and reliable estimates of pile capacity. This approach enhances the accuracy of predictions, supports safer foundation designs, and promotes cost efficiency.

Furthermore, the Bayesian method's ability to iteratively update soil parameter estimates as new data becomes available makes it particularly advantageous for complex and heterogeneous soil conditions such as those found in tropical coastal areas like Bali. Overall, incorporating probabilistic methods like Bayesian analysis in foundation design represents a promising advancement toward improved geotechnical engineering practices that accommodate soil variability and uncertainty, ensuring both structural safety and economic optimization.

5. REFRENCES

Albert, J., & Hu, J. (2019). *Probability and Bayesian Modeling* (1st ed.). Chapman and Hall/CRC. https://doi.org/https://doi.org/10.1201/9781351030144

Allenby, G. M., Rossi, P. E., & Mcculloch, R. E. (2005). *Hierarchical Bayes Models: A Practitioners Guide Hierarchical Bayes Models*.

Baecher, G. B. (2017). Bayesian thinking in geotechnics. *Geo Risk*, 1–18. https://doi.org/10.1061/9780784480694.001

Basoka, I. W. A., Kasama, K., Furukawa, Z., & Rifa'i, A. (2023, December). Bayesian hierarchical model on crushability of pumice particle strength. *Geotechnics for Sustainable Infrastructure Development - Geotec Hanoi*.

Basoka, I. W. A., Kasama, K., Furukawa, Z., & Rifa'i, A. (2024). Bayesian Hierarchical Model on Crushability of Pumice Particle Strength. In P. Duc Long & N. T. Dung (Eds.), *Proceedings of the 5th International Conference on Geotechnics for Sustainable Infrastructure Development* (pp. 1871–1884). Springer Nature Singapore.

Bozorgzadeh, N., Harrison, J. P., & Escobar, M. D. (2019). Hierarchical Bayesian modelling of geotechnical data: Application to rock strength. *Geotechnique*, *69*(12), 1056–1070. https://doi.org/10.1680/jgeot.17.P.282

Britten, G. L., Mohajerani, Y., Primeau, L., Aydin, M., Garcia, C., Wang, W. L., Pasquier, B., Cael, B. B., & Primeau, F. W. (2021). Evaluating the Benefits of Bayesian Hierarchical Methods for Analyzing Heterogeneous Environmental Datasets: A Case Study of Marine Organic Carbon Fluxes. *Frontiers in Environmental Science*, *9*. https://doi.org/10.3389/fenvs.2021.491636

Contreras, L. F., Brown, E. T., & Ruest, M. (2018). Bayesian data analysis to quantify the uncertainty of intact rock strength. *Journal of Rock Mechanics and Geotechnical Engineering*, *10*(1), 11–31. https://doi.org/10.1016/j.jrmge.2017.07.008

Feng, Y., Gao, K., Mignan, A., & Li, J. (2021). Improving local mean stress estimation using Bayesian hierarchical modelling. *International Journal of Rock Mechanics and Mining Sciences*, *148*. https://doi.org/10.1016/j.ijrmms.2021.104924

Fornacon-Wood, I., Mistry, H., Johnson-Hart, C., Faivre-Finn, C., O'Connor, J. P. B., & Price, G. J. (2022). Understanding the Differences Between Bayesian and Frequentist Statistics. In *International Journal of Radiation Oncology Biology Physics* (Vol. 112, Issue 5, pp. 1076–1082). Elsevier Inc. https://doi.org/10.1016/j.ijrobp.2021.12.011

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., Rubin, D. B., Carlin, J., Stern, H., Rubin, D., & Dunson, D. (2021). *Bayesian Data Analysis* (3rd ed.).

Jiao, Y., Cortés, E., Andrews, K., & Guo, F. (2011). Poor-data and data-poor species stock assessment using a Bayesian hierarchical approach. *Ecological Applications*, *21*(7), 2691–2708. https://doi.org/10.1890/10-0526.1

Kelly, R., & Huang, J. (2015). Bayesian updating for one-dimensional consolidation measurements. *Canadian Geotechnical Journal*, *52*(9), 1318–1330. https://doi.org/10.1139/cgj-2014-0338

Kruschke, J. K. (2021). Bayesian Analysis Reporting Guidelines. *Nature Human Behaviour*, *5*(10), 1282–1291. https://doi.org/10.1038/s41562-021-01177-7

Li, T., Griffiths, W. D., & Chen, J. (2017). Weibull Modulus Estimated by the Non-linear Least Squares Method: A Solution to Deviation Occurring in Traditional Weibull Estimation. *Metallurgical and Materials Transactions A: Physical Metallurgy and Materials Science*, *48*(11), 5516–5528. https://doi.org/10.1007/s11661-017-4294-4

Phoon, K. K., Cao, Z. J., Ji, J., Leung, Y. F., Najjar, S., Shuku, T., Tang, C., Yin, Z. Y., Ikumasa, Y., & Ching, J. (2022). Geotechnical uncertainty, modeling, and decision making. *Soils and Foundations*, *62*(5). https://doi.org/10.1016/j.sandf.2022.101189

Rossi, R. J. (2018). *Mathematical Statistics : An Introduction to Likelihood Based Inference*. John Wiley & Sons.

Ueda, K. (2022). Hierarchical Bayesian approach to estimating variability of liquefaction resistance of sandy soils considering individual differences in laboratory tests. *Journal of Geotechnical and Geoenvironmental Engineering*, *148*(2). https://doi.org/10.1061/(asce)gt.1943-5606.0002749

Wakefield, J. (2013). *Bayesian and frequentist regression methods* (1st ed.). Springer New York. https://doi.org/10.1007/978-1-4419-0925-1

Zaidi, A., Bouamama, B. O., & Tagina, M. (2012). Bayesian reliability models of weibull systems: State of the art. *International Journal of Applied Mathematics and Computer Science*, *22*(3), 585–600. https://doi.org/10.2478/v10006-012-0045-2